F19PL Abstract Algebra

COURSE DETAILS
Course Code: F19PL
Full Course Title: Abstract Algebra
SCQF Level: 9
SCAF Credits: 15
Available as Elective: No

DELIVERY LEVEL
Undergraduate: Yes  Postgraduate Taught: Yes  Postgraduate Research: No

Additional Information:

COURSE AIMS
The objective of the module is to introduce and develop the concepts and methods of abstract or structural algebra, with emphasis on its generality and widespread applications, and on the efficacy of the axiomatic method and abstract reasoning.

LEARNING OUTCOMES – SUBJECT MASTERY
By the end of the course, students should be able to:

- give standard examples of groups, rings and fields
- carry out simple deductions using axioms for groups, rings, and fields
- understand the notion of rings and fields, and the properties of Euclidean rings
- understand the construction of a quotient ring
- apply the first isomorphism theorem to prove that two rings or fields are isomorphic
- understand polynomial rings and their use to construct finite fields
- understand the significance of the properties of rings in solving certain diophantine equations
- perform standard group computations with linear isometries of the plane and permutations of a finite set
- construct and complete Cayley tables for small groups
- understand the notion of subgroup and determine whether a given subset is a subgroup
- apply Lagrange's theorem to study the subgroups of a given group
- understand the notions of homomorphism and isomorphism in groups, rings, and fields
- understand how group actions can be used to obtain new information about a group / a set.
- understand the notion of automorphism group of a field, and how this notion is related to the solubility of polynomial equations

LEARNING OUTCOMES – PERSONAL ABILITIES

- Demonstrate the ability to learn independently
- Demonstrate knowledge of an area of mathematics.
- Manage time, work to deadlines and prioritise workloads

SYLLABUS
Rings and fields: Subrings and subfields of \( \mathbb{C} \). Axioms for rings and fields. Polynomial rings and rings of integers modulo \( n \). Subrings and subfields.

Divisibility in commutative rings: Euclidean rings and their ideals. Unique factorisation in Euclidean rings. Application to integers that can be written as a sum of two squares.

Homomorphisms and quotient rings: Ring homomorphisms. Quotient rings and the First Isomorphism Theorem. Application to quotients of polynomial rings. Finite fields.

Groups of transformations: Group of symmetries of an equilateral triangle. Transformation groups. First look at the symmetric group: Matrix and cycle decomposition for permutations.

Abstract groups: Axioms for a group and consequences. Cayley tables. Groups coming from sets of numbers, from modular arithmetic, and from matrices.


The unsolvability of the quintic: Algebraic numbers and algebraic number fields. Galois groups and soluble groups. The unsolvability of the quintic.