F19NB Numerical Analysis B

COURSE DETAILS

Course Code: F19NB
Full Course Title: Numerical Analysis B
SCQF Level: 9
SCAF Credits: 15
Available as Elective: No

DELIVERY LEVEL

Undergraduate: Yes  Postgraduate Taught: Yes  Postgraduate Research: No

Additional Information:

COURSE AIMS

The course aims to provide an introduction to function approximation and interpolation methods in 1- and 2-D; to continue the study of numerical integration methods; to study the techniques required to apply and analyse numerical methods for solving linear systems of equations and eigenvalue problems. By the end of the course, students should be able to apply the methods and algorithms listed below, and carry out the associated analysis of errors, convergence and operations counts.

LEARNING OUTCOMES – SUBJECT MASTERY

By the end of the course, students should be able to:

- write Python procedures for the methods considered in the course.
- understand linear systems of equations, vectors, matrices.
- understand how to count FLOPS in algorithms.
- understand block, sparse, lower and upper triangular matrices and memory requirements.
- understand how to multiply matrices.
- understand Gaussian elimination and estimate FLOPS.
- understand the need for pivoting for reducing round-off errors in GE algorithms.
- understand how to compute the matrix inverse by the companion matrix approach and estimate FLOPS.
- understand the sparse 1d and 2d Laplacian matrices.
- understand theLUdecomposition and its relationship with GE and estimate FLOPS.
- understand how to compute the determinant from theLUdecomposition.
- understand theLDL×decomposition for symmetric matrices and how to compute it from GE.
- understand theLL×decomposition for symmetric and positive definite matrices and how to compute it from theLDL×decomposition.
- understand orthonormal basis, columnwise orthonormal matrices, the Gram-Schmidt procedure and estimate FLOPS.
- understand how to compute theQRdecomposition incrementally from the Gram-Schmidt procedure and estimate FLOPS.
- understand how to solve overdetermined problems in the least-squares sense using theQRdecomposition.
- understand the general ideas behind iterative algorithms for the solution of linear systems.
- know the Jacobi, Gauss-Seidel and Successive Over-Relaxation methods and analysis in terms of the iteration matrix.
- understand eigenvalues and eigenvectors.
- understand the GMRES algorithm, be able to compute GMRES iterates and analyze the convergence and the FLOPS count.
- understand the power method, the shifted inverse power method and be able to analyze the convergence of
these algorithms and the FLOPS count.

- understand deflation for finding the eigenvalues of block upper-triangular matrices.
- understand how to compute the conjugate $Q^{-1}AQ$ of a matrix and use it to find eigenvalues.
- understand reduction to upper Hessenberg form using Householder reflections.
- understand deflation for reducible upper Hessenberg matrices.
- understand the Francis QR iteration for calculating eigenvalues, including Householder reflections.

Appreciate the role of error estimates.

Appreciate direct and iterative methods for solving linear systems.

Appreciate the value of careful analysis of algorithms for efficiency and accuracy.

**LEARNING OUTCOMES – PERSONAL ABILITIES**

Use logical reasoning to prove theorems

Organize complex calculations in a clear manner

Be able to write simple programmes in Python.

Be aware of the importance of understanding and analysing errors.

Be able to present a written account of technical material.

**SYLLABUS**


Matrix decompositions: $QR$, $LU$, $LL^*$ and $LDL^*$ decomposition. Operation counts. Least squares problems.
Iterative algorithms for the solution of linear systems: Jacobi, Gauss-Seidel, GMRES. Convergence analysis.


**COURSE RELATIONSHIPS**

<table>
<thead>
<tr>
<th>Course Code</th>
<th>Level</th>
<th>Title</th>
<th>School</th>
<th>Type</th>
</tr>
</thead>
<tbody>
<tr>
<td>F18CF</td>
<td>8</td>
<td>Linear Algebra</td>
<td>School of Math and Comp Sci.</td>
<td>Pre-Requisite</td>
</tr>
</tbody>
</table>

**LOCATION AND ASSESSMENT METHODS**

<table>
<thead>
<tr>
<th></th>
<th>Edi</th>
<th>SBC</th>
<th>Ork</th>
<th>Dub</th>
<th>Malay</th>
<th>IDL</th>
<th>COLL</th>
<th>ALP</th>
<th>OTH</th>
<th>Method</th>
<th>Weight</th>
<th>Exam Mins</th>
<th>Type</th>
<th>Diet</th>
<th>Synoptic Course</th>
</tr>
</thead>
<tbody>
<tr>
<td>Y</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td>Examination</td>
<td>90</td>
<td>120</td>
<td>Assessment</td>
<td>Semester 2</td>
<td></td>
</tr>
<tr>
<td>Y</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td>Coursework</td>
<td>10</td>
<td></td>
<td>Assessment</td>
<td>Semester 2</td>
<td></td>
</tr>
<tr>
<td>Y</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td>Examination</td>
<td>100</td>
<td></td>
<td>Reassessment</td>
<td>Semester 3</td>
<td></td>
</tr>
</tbody>
</table>