F18CE Multivariable Calculus and Real Analysis B

COURSE DETAILS
Course Code: F18CE
Full Course Title: Multivariable Calculus and Real Analysis B
SCQF Level: 8
SCAF Credits: 15
Available as Elective: No

DELIVERY LEVEL
Undergraduate: Yes  Postgraduate Taught: Yes  Postgraduate Research: No

Additional Information:

COURSE AIMS
The course aims to introduce students to the idea of rigorous mathematical arguments and, in particular, to discuss the rigorous foundations of calculus. An important feature of the course is the use of careful, rigorous proofs of the theorems used and one of the aims of the course is to improve student's ability to understand such arguments and to develop such proofs for themselves. A central concept in analysis is the idea of convergence, either of sequences, series or of functions, and this course aims to introduce this concept and provide the basic results which will be used in later courses. In addition, it will give methods of obtaining inequalities and approximations (with precise estimates of how good the approximations are), tests for convergence of series and power series and ways of identifying functions defined by power series and characterisations of functions (over bounded and unbounded intervals) for which the concept of area under the graph of a function makes sense.

LEARNING OUTCOMES – SUBJECT MASTERY
By the end of the course, students should be able to:

- know what a sequence of numbers is.
- be able to prove from the definition that a sequence of numbers is bounded.
- understand the definition of convergence of a sequences of numbers, and be able to prove from the definition that simple sequences of numbers are convergent.
- prove results on combinations of sequences.
- prove that a convergent sequence is bounded.
- prove results on inequalities satisfied by limits, e.g., \( an < a \Rightarrow \lim_{n \to \infty} an \leq a \).
- understand the definition of the sup and inf of a set of numbers.
- understand the completeness axiom for the set of real numbers.
- be able to prove from the definition that a set of numbers is bounded above or below.
- understand the idea of monotone sequences and the monotone convergence theorem.
- use the monotone convergence theorem to show that sequences are convergent, without knowing the limit. Use this to retrospectively find the limit. In particular, to do this for iteratively defined sequences.
- understand the idea of subsequences and construct examples illustrating the behaviour of subsequences.
- state the Bolzano-Weierstrass theorem, and use it to show that sequences have convergent subsequences.
- limits of functions
- understand the limit and the \( \varepsilon \)-definition of continuity of functions.
- prove simple results involving continuous functions and their combinations.
- determine if given functions are continuous or discontinuous.
- prove that a continuous function on a closed interval is bounded and attains its bounds.
- prove the intermediate value theorem.
- use the intermediate value theorem to prove that certain equations have solutions in appropriate intervals.
LEARNING OUTCOMES – PERSONAL ABILITIES

• Demonstrate the ability to learn independently
• Demonstrate knowledge of an area of mathematics.
• Manage time, work to deadlines and prioritise workloads

SYLLABUS

Sequences: Briefly recall the idea of a sequence of real numbers. Bounded and convergent sequences, and the definition of the limit of a sequence. General theorems about limits.

Suprema and infima: Sup and inf of sets of real numbers. The completeness axiom for real numbers.

Monotone sequences: Monotone sequences and the monotone convergence theorem. Use of the monotone convergence theorem to prove convergence of sequences without knowing the limit.

Subsequences and the Bolzano-Weierstrass theorem.

Continuous functions: Limits of functions, one-sided limits. General theorems about limits of functions. Continuity, combinations of continuous functions. Boundedness of continuous functions on closed intervals. The intermediate value theorem. Uniform continuity

First mean value theorem: Statement and proof of the first mean value theorem, and applications to inequalities

Series and power series: Convergence of series. The comparison, ratio, zero, absolute convergence and alternating series tests for series. Power series, and the radius of convergence of a power series

Riemann integration and convergence of integrals: Partitions, upper and lower sums, Riemann integrable functions.

### COURSE RELATIONSHIPS

<table>
<thead>
<tr>
<th>Course Code</th>
<th>Level</th>
<th>Title</th>
<th>School</th>
<th>Type</th>
</tr>
</thead>
<tbody>
<tr>
<td>F17CA</td>
<td>7</td>
<td>Calculus A</td>
<td>School of Math and Comp Sci.</td>
<td>Pre-Requisite</td>
</tr>
<tr>
<td>F17CB</td>
<td>7</td>
<td>Calculus B</td>
<td>School of Math and Comp Sci.</td>
<td>Pre-Requisite</td>
</tr>
<tr>
<td>F18CD</td>
<td>8</td>
<td>Multivariable Calculus and Real Analysis A</td>
<td>School of Math and Comp Sci.</td>
<td>Linked</td>
</tr>
</tbody>
</table>

### LOCATION AND ASSESSMENT METHODS

<table>
<thead>
<tr>
<th>Edi</th>
<th>SBC</th>
<th>Ork</th>
<th>Dub</th>
<th>Malay</th>
<th>IDL</th>
<th>COLL</th>
<th>ALP</th>
<th>OTH</th>
<th>Method</th>
<th>Weight</th>
<th>Exam Mins</th>
<th>Type</th>
<th>Diet</th>
<th>Synoptic Course</th>
</tr>
</thead>
<tbody>
<tr>
<td>Y</td>
<td>Y</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td>Examination</td>
<td>85</td>
<td>120</td>
<td>Assessment</td>
<td>Semester 2</td>
<td></td>
</tr>
<tr>
<td>Y</td>
<td>Y</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td>Coursework</td>
<td>15</td>
<td></td>
<td>Assessment</td>
<td>Semester 2</td>
<td></td>
</tr>
<tr>
<td>Y</td>
<td>Y</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td>Examination</td>
<td>100</td>
<td>120</td>
<td>Reassessment</td>
<td>Semester 3</td>
<td></td>
</tr>
</tbody>
</table>