COURSE DETAILS

Course Code: F17CC
Full Course Title: Introduction to University Mathematics
SCQF Level: 7
SCAF Credits: 15
Available as Elective: No

DELIVERY LEVEL

Undergraduate: Yes  Postgraduate Taught: No  Postgraduate Research: No

COURSE AIMS

The course aims to provide a bridge between school and university (mainly, non-calculus) mathematics.

It will provide an introduction to the culture of mathematics including discussions of history, modern applications and the nature of reasoning, problem-solving and proofs.

In addition, basic skills will be developed in elementary combinatorics, complex numbers and polynomials, the algebra of matrices and their applications and geometry and vectors.

LEARNING OUTCOMES – SUBJECT MASTERY

By the end of the course, students should be able to:

- carry out all the procedures described in the syllabus;
- be able to read and construct simple proofs;
- understand how and why mathematics at university is different from mathematics at school

LEARNING OUTCOMES – PERSONAL ABILITIES

- Demonstrate the ability to learn independently
- Demonstrate knowledge of an area of mathematics.
- Manage time, work to deadlines and prioritise workloads

SYLLABUS

The conceptual aspects of mathematics:

- What is mathematics? Mathematics in history and contemporary mathematics.
- Reasoning and logic. What is an argument? The notion of proof in mathematics with simple first examples such as the irrationality of \(\sqrt{2} \), the triangle theorem, Pythagoras’ theorem, and the proof that there are infinitely many
primes.

- Abstraction and rules. The meaning of key algebraic terms such as: associativity, commutativity, distributivity, identity, inverse. The rules of high-school algebra. Proof that \(-1 \times -1 = 1\).
- Problem-solving.
- The need for checking.

1. Combinatorics:

- Counting
  - Manipulate sets and their elements. This includes the Boolean operations and set product.
  - Answer simple counting questions involving permutations and combinations. Connections with probability touched on.
  - Statement and proof of the Binomial theorem (by a counting argument). Applications.

2. Complex numbers and polynomials:

- How complex numbers were discovered.
- Add, subtract, multiply and divide complex numbers.
- Find square roots of complex numbers.
- Solve quadratics by completing the square.
- Represent complex numbers in the complex plane.
- Understand the geometric interpretations of addition and multiplication of complex numbers.
- Proof that a polynomial of degree \(n\) over the complex numbers has at most \(n\) roots.
- The fundamental theorem of algebra.
- Proof of the fundamental theorem of algebra for real polynomials.
- Find \(n\)th roots.
- Use De Moivre's theorem to find expressions for \(\sin \theta\) and \(\cos \theta\).
- Euler's theorem and its proof.
- Find rational roots of polynomials with integer coefficients.
- Factorize real and complex polynomials appropriately.
- Understand the difference between trigonometric solutions and radical solutions.

3. Matrices:

- Why matrices are important.
• Add, subtract, and multiply two matrices, and multiply a matrix by a scalar; be able to carry out sequences of such operations to obtain a single matrix as a result. The main emphasis will be on 'small' matrices often 2×2 or 3×3 throughout.
• Proof of associativity for matrix multiplication.
• Solve linear equations using Gaussian elimination.
• Proof of the fundamental theorem of linear equations.
• Compute determinants by first row expansion.
• Compute matrix inverses using the adjugate method.
• Calculate the characteristic polynomial of a matrix.
• Statement of the Cayley-Hamilton theorem and proof in the 2×2 case.

4. Vectors:

• What is Euclidean geometry?
• Compute with vectors using inner products, vector products, and scalar triple products.
• Find the equation of the unique line determined by two points or a point and a vector in space.
• Find the equation of the unique plane determined by three points or by a point and a normal.
• Calculate intersections of lines or planes.
• Derivation of the volume of a parallelepiped using scalar triple products and connection with determinants.

### COURSE RELATIONSHIPS

N/A

### LOCATION AND ASSESSMENT METHODS

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