COURSE DETAILS
Course Code: F17CC
Full Course Title: Introduction to University Mathematics
SCQF Level: 7
SCAF Credits: 15
Available as Elective: No

DELIVERY LEVEL
Undergraduate: Yes
Postgraduate Taught: No
Postgraduate Research: No
Additional Information:

COURSE AIMS
The course aims to provide a bridge between school and university (mainly, non-calculus) mathematics.

It will provide an introduction to the culture of mathematics including discussions of history, modern applications and the nature of reasoning, problem-solving and proofs.

In addition, basic skills will be developed in elementary combinatorics, complex numbers and polynomials, the algebra of matrices and their applications and geometry and vectors.

LEARNING OUTCOMES – SUBJECT MASTERY
By the end of the course, students should be able to:

• carry out all the procedures described in the syllabus;
• be able to read and construct simple proofs;
• understand how and why mathematics at university is different from mathematics at school

LEARNING OUTCOMES – PERSONAL ABILITIES

• Demonstrate the ability to learn independently
• Demonstrate knowledge of an area of mathematics.
• Manage time, work to deadlines and prioritise workloads

SYLLABUS
The conceptual aspects of mathematics:

• What is mathematics? Mathematics in history and contemporary mathematics.
• Reasoning and logic. What is an argument? The notion of proof in mathematics with simple first examples such as the irrationality of \( \sqrt{2} \), the triangle theorem, Pythagoras' theorem, and the proof that there are infinitely many
primes.

• Abstraction and rules. The meaning of key algebraic terms such as: associativity, commutativity, distributivity, identity, inverse. The rules of high-school algebra. Proof that $-1 \times -1 = 1$.
• Problem-solving.
• The need for checking.

1. Combinatorics:

• Counting
• Manipulate sets and their elements. This includes the Boolean operations and set product.
• Answer simple counting questions involving permutations and combinations. Connections with probability touched on.
• Statement and proof of the Binomial theorem (by a counting argument). Applications.

2. Complex numbers and polynomials:

• How complex numbers were discovered.
• Add, subtract, multiply and divide complex numbers.
• Find square roots of complex numbers.
• Solve quadratics by completing the square.
• Represent complex numbers in the complex plane.
• Understand the geometric interpretations of addition and multiplication of complex numbers.
• Proof that a polynomial of degree $n$ over the complex numbers has at most $n$ roots.
• The fundamental theorem of algebra.
• Proof of the fundamental theorem of algebra for real polynomials.
• Find $n$th roots.
• Use De Moivre's theorem to find expressions for $\sin n\theta$ and $\cos n\theta$.
• Euler's theorem and its proof.
• Find rational roots of polynomials with integer coefficients.
• Factorize real and complex polynomials appropriately.
• Understand the difference between trigonometric solutions and radical solutions.

3. Matrices:

• Why matrices are important.
• Add, subtract, and multiply two matrices, and multiply a matrix by a scalar; be able to carry out sequences of such operations to obtain a single matrix as a result. The main emphasis will be on ‘small’ matrices often 2×2 or 3×3 throughout.
• Proof of associativity for matrix multiplication.
• Solve linear equations using Gaussian elimination.
• Proof of the fundamental theorem of linear equations.
• Compute determinants by first row expansion.
• Compute matrix inverses using the adjugate method.
• Calculate the characteristic polynomial of a matrix.
• Statement of the Cayley-Hamilton theorem and proof in the 2×2 case.

4. Vectors:

• What is Euclidean geometry?
• Compute with vectors using inner products, vector products, and scalar triple products.
• Find the equation of the unique line determined by two points or a point and a vector in space.
• Find the equation of the unique plane determined by three points or by a point and a normal.
• Calculate intersections of lines or planes.
• Derivation of the volume of a parallelepiped using scalar triple products and connection with determinants.

COURSE RELATIONSHIPS

N/A

LOCATION AND ASSESSMENT METHODS

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