F11ND Numerical Analysis (PDEs)

COURSE DETAILS
Course Code: F11ND
Full Course Title: Numerical Analysis (PDEs)
SCQF Level: 11
SCAF Credits: 15
Available as Elective: No

DELIVERY LEVEL
Undergraduate: Yes
Postgraduate Taught: Yes
Postgraduate Research: No

COURSE AIMS
To provide an introduction to the techniques and analysis required to find the numerical solution of partial differential equations using both finite difference and finite element approaches and to provide practical experience of implementing the techniques in practical assignments

LEARNING OUTCOMES – SUBJECT MASTERY
By the end of the course, students should be able to:

- construct finite differences for approximating partial derivatives
- analyse the local truncation error using Taylor series expansions
- perform stability analysis and state the Lax equivalence theorem
- write finite difference schemes in matrix algebra form
- derive the Euler, backward Euler and \( \Theta \)-methods for solving parabolic PDE's
- distinguish explicit and implicit schemes
- apply the fictitious point method for incorporating flux boundary conditions
- apply ADI schemes for treating parabolic PDE's with more than one space dimension
- perform phase lag analysis for numerical solution of hyperbolic PDE's
- derive the leapfrog, Lax-Wendroff and Crank-Nicolson schemes for solving the advection equation
- use nonlinear conservation laws to derive the switching and nonlinear Lax-Wendroff schemes
- understand the variational formulation of the finite element method
- calculate linear and bilinear nodal basis functions for the FEM
- use the basis functions to calculate matrix elements for the FEM
- write computer code to approximate the solution of PDEs

LEARNING OUTCOMES – PERSONAL ABILITIES

- Demonstrate the ability to learn independently
- Demonstrate knowledge of an area of mathematics.
- Manage time, work to deadlines and prioritise workloads

SYLLABUS
F11ND Numerical Analysis (PDEs)

Finite difference approximations: classification of PDE's; forward, backward and central differences, Taylor series.

Parabolic PDEs: finite difference approximation of the heat equation; local truncation error analysis; stability and convergence; multi-level and ADI schemes.

Hyperbolic PDEs: travelling wave solutions; comparison of schemes for the advection equation; second order wave equations; nonlinear conservation laws.

Elliptic PDEs: Introduction, simple finite difference scheme. Finite element method (FEM); variational formulation of the FEM; nodal basis functions and matrix elements.

Revision and problem solving

COURSE RELATIONSHIPS

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<th>Course Code</th>
<th>Level</th>
<th>Title</th>
<th>School</th>
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LOCATION AND ASSESSMENT METHODS

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